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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

A scheme is developed for commanding a spacecraft to perform a precise, predetermined maneuver. The example considered is an Advanced Orbiting Solar Observatory performing a raster scan over a small celestial area. During the scan, the position error is required to be less than a prescribed amount at any given time. To satisfy this requirement the transition between scan lines must be smooth to avoid both velocity and position errors at the start of the subsequent scan. To accomplish a smooth transition the state at the ends of the scan line is specified and the variational form of optimal control theory is used to find the command that will use the minimum amount of energy. The form of the command from this solution was also used for the scan lines when the trajectory was completely specified. It is possible to apply this technique to any given maneuver composed of specified trajectory segments and to obtain a smooth transition from segment to segment. The theoretical results were developed for small angle motions of a single axis controller and, therefore, apply for motion within a small area only.

The scheme was tested by having it control a table mounted on a spherical gas bearing. The control torque was generated by reaction wheels, and the command was calculated by a digital computer. The test maneuver was a simple repetitive scan along two parallel lines.

INTRODUCTION

Some spacecraft missions require that a sensor or other instrument traverse a precise path over a region of the celestial sphere. For such missions it may be convenient to fix the instrument to the spacecraft and maneuver the entire vehicle, as has been proposed for the Advanced Orbiting Solar Observatory (AOSO). The instruments of this spacecraft must precisely scan a specific area near or within the solar disk. To do this the vehicle moves along a series of parallel lines. A transition maneuver transfers the vehicle from one line to another. The control problem is to design a system which will smoothly and efficiently drive the vehicle through such a segmented maneuver and maintain the required position tolerance.

One approach applied to generating the single axis motion along an AOSO scan line is presented in reference 1. The controller, which was a part of the control loop, was of several different nonlinear forms and contained

relays or proportional elements with saturation. The reason for choosing a relay controller was to take advantage of the ideas of minimum time optimal control and both the relay and the proportional elements with saturation were used to control the vehicle while it moved along a scan line. The control was designed to complete the transition maneuver in a minimum time. The controller therefore used the maximum available torque and adjusted the duration according to the wheel speed to provide a smooth transition to the succeeding scan. This turn-around is essentially an open-loop control and depends strongly on a precise knowledge of the system characteristics for proper operation.

This report describes another approach in which the spacecraft control always operates as a closed-loop system that follows a signal calculated by the controller. The characteristics of this system are fixed and independent of the maneuver required of the vehicle. The control loop is assumed to consist of a reaction wheel drive with feedback proportional to position plus rate. The controller is therefore required to generate the necessary input to this control loop to drive the vehicle with the required performance tolerance. When a maneuver segment is specified by only the end points of the trajectory, the variational form of optimal control theory is used to solve for the necessary form of the controller. The same form of controller input is used for maneuver segments when the trajectory is completely specified so that the transition between segments will be smooth. The analysis is limited to a small angle motion about a single axis. For maneuvers involving rotations about more than one axis, the single axis trajectories are superposed.

As a final part of the study, the command scheme was tested experimentally. In the simulation, a table mounted on a spherical gas bearing was driven by reaction wheels. Optical sensors and rate gyros were used to sense position and rate. The command signal was generated by an external digital computer.

LIST OF SYMBOLS

e   input voltage to reaction wheel motors

$E(s)$ Laplace transform of ϵ

H   total angular momentum of vehicle and wheel

i   reaction wheel motor current

I   vehicle moment of inertia without reaction wheels

J   reaction wheel moment of inertia about spin axis

K   motor torque gain

K_m motor back-emf gain

2

K_P	torque gain for pitch
K_Y	torque gain for yaw
K_α	position feedback gain
$K_{\dot{\alpha}}$	rate feedback gain
K_θ	pitch position gain
$K_{\dot{\theta}}$	pitch rate gain
K_ψ	yaw position gain
$K_{\dot{\psi}}$	yaw rate gain
l	cost function integrand
L	cost function
R	motor winding resistance
t	time
T	control time
T_e	external torques
u	control input
$\bar{x}$	state vector
x_1, x_2, x_3	state vector components
$\dot{\bar{x}}$	$d\bar{x}/dt$
$X(S)$	Laplace transform of x
α	position angle
$\dot{\alpha}$	$d\alpha/dt$
$\ddot{\alpha}$	$d^2\alpha/dt^2$
$\epsilon, \dot{\epsilon}, \ddot{\epsilon}$	position, rate, and acceleration errors
$\theta, \dot{\theta}$	pitch angle and rate
$\bar{\lambda}$	adjoint state vector

$\dot{\bar{\lambda}}$ $d\bar{\lambda}/dt$

$\lambda_1, \lambda_2, \lambda_3$ adjoint vector components

$\psi, \dot{\psi}$ yaw angle and rate

ω reaction wheel speed measured relative to vehicle

$\dot{\omega}$ $d\omega/dt$

Ω vehicle angular rate with respect to inertial space

$\partial/\partial u$ gradient with respect to u

$|\partial/\partial x|'$ gradient with respect to x transposed

ANALYSIS

General Problem Description

The problem is to find the command that will force a satellite with some sort of control system to perform a programmed maneuver within specified constraints. It is assumed that the characteristics of the control system and satellite are fixed, so that the problem is to design a controller that will supply the required commands. A block diagram of the complete system is shown in figure 1.

The precise raster-scan motion of an AOSO will serve as an example of a maneuver with a specific repeating pattern over a region of small angular motion. Two general types of motion are required of each axis of the vehicle. The first is a specified trajectory given as a function of time. The other is a motion between end points with the time of transfer but not the trajectory specified. For an AOSO scan, the motion along the scan line would be of the type when the trajectory is specified, and the transfer between lines would be of the end-point type. The end-point control problem can be handled by optimal control theory; the variational form of optimal control theory with a minimum energy constraint will be used to solve this problem. As a means of providing a smooth transition between maneuver segments the basic form of the control that results from the optimal control theory solution will be adapted to the specific trajectory portion of the motion. The control solution for the entire scan motion can then be made up of a sequence of commands of the appropriate type.

In the analysis, a number of engineering choices were made as a consequence of using optimal control theory to solve the problem of transfer from one state point to another. The first choice was the form of optimal control theory to be used. To avoid the use of on-off controller inputs the variational form of optimal control theory was chosen. This means that no bound was assumed on the control variable. The second choice was the cost

function (minimizing time or energy, and possibly some measure of the system states) for use with the theory. Since the type of maneuver being considered is to be done in fixed time, the choice was to minimize energy expended in the reaction wheels for a given maneuver. This cost would relate directly to the overall weight of the spacecraft through battery size and solar panels required. Third, there was the choice of including the position and rate feedback gains as a basic part of the vehicle control system. For the satellite problem being considered, the basic plant components are the moment of inertia of the vehicle and the reaction wheel torquers. If optimal control theory with the minimum energy cost is applied to this plant, the results will generally be of the form of either an open-loop command to the torquers or a time varying feedback to be incorporated as a part of the plant. Solutions of this sort can be seen in many books on optimal control theory. However, the open-loop-type command is unable to cope with a changing environment and is sensitive to plant characteristics that are poorly defined or only approximately described by the mathematics. Also a time varying feedback usually requires an infinite gain at the end of the maneuver. To avoid these problems and to design a system that was practical from an engineering point of view, it was decided to use the open-loop-type approach but to define the plant as a closed-loop system that was inherently stable. It was therefore assumed that there would be a proportional position plus rate feedback in operation for each axis of the satellite and the command for the maneuver would be an external input to this system. The feedbacks allow the vehicle to track the command trajectory despite external disturbances and irregularities in component characteristics.

Plant Equations

Figure 1 shows the items considered as a part of the plant to be controlled. It is assumed that the feedback loops shown in the figure exist; the values of the gains are not as yet specified. The problem is to be considered one axis at a time and the following analysis is for a typical axis. The vehicle torquing is to be done by reaction wheels, the vehicle position is measured by optical sensors, and the vehicle angular rate is measured by rate gyros. The equations of motion are derived in appendix A.

The equations of rotational motion about one axis are (eq. (A6))

$$I\ddot{\alpha} + J(\ddot{\alpha}+\dot{\omega}) = T_e \tag{1a}$$

$$J(\ddot{\alpha}+\dot{\omega}) = \frac{K}{R}(K_\alpha\alpha+K_{\dot{\alpha}}\dot{\alpha}+u-K_m\omega) \tag{1b}$$

The variable α is the angular position of the vehicle; ω is the reaction wheel speed relative to the vehicle; and T_e represents some external torque to the system. The remaining constants pertain to feedback gains and other vehicle characteristics.

For equations (1a) and (1b) to be used with optimal control theory as normally formulated, they must be converted to the state space matrix form

$$\dot{\overline{x}} = F\overline{x} + Du \tag{2}$$

Three states are necessary and are defined as follows:

$$\left.\begin{array}{l} x_1 = \alpha \\[4pt] x_2 = \dot{\alpha} \\[4pt] x_3 = H \end{array}\right\} \tag{3}$$

where H is the total angular momentum of the vehicle and reaction wheel system. If the external torques are assumed to be zero, the total angular momentum will then be a constant and the plant equation can be written as

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -\dfrac{K}{R}\dfrac{K_\alpha}{I} & -\left(\dfrac{KK_{\dot\alpha}}{RI} + \dfrac{KK_m}{RI}\ \dfrac{I+J}{J}\right) & \dfrac{KK_m}{RIJ} \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 \\ -\dfrac{K}{RI} \\ 0 \end{vmatrix} u \tag{4}$$

These equations are the same as equations (1) but written as a set of first-order differential equations.

CONTROLLER EQUATIONS

End-Point Control

The end-point controller problem is to find an input (u) to the plant that will transfer the system state from one point to another with a minimum power (expended in the reaction wheel motors). The initial states $\overline{x}(0)$ and the final states $\overline{x}(f)$ are given and also the time T during which the transfer is to be made. The problem then falls into the category of a fixed time, fixed end-point problem with no bound on the control variable. The mathematical formulation of the problem is as follows.

Plant equation: $$\dot{\overline{x}} = F\overline{x} + Du \tag{5a}$$

Boundary conditions: $$\left.\begin{array}{l} \overline{x} = \overline{x}(0) \quad \text{at} \quad t = 0 \\[6pt] \overline{x} = \overline{x}(f) \quad \text{at} \quad t = T \end{array}\right\} \tag{5b}$$

Cost function: $$L = \int_0^T l\ dt = \int_0^T (i^2 R) dt \tag{5c}$$

The problem is to determine (u) to minimize L.

The solution to the above problem is given in various forms in the literature on optimal control. Reference 2 shows a typical development where a necessary condition for the solution to the problem is that the following set of matrix differential equations are satisfied.

Plant equation:
$$\dot{\overline{x}} = F\overline{x} + Du \qquad (6a)$$

Adjoint equation:
$$\dot{\overline{\lambda}} = -\left|\frac{\partial l}{\partial \overline{x}}\right|' - \left|\frac{\partial(F\overline{x}+Du)}{\partial \overline{x}}\right|'\overline{\lambda} \qquad (6b)$$

Control equation:
$$\frac{\partial}{\partial u}\left[l + \overline{\lambda}'(F\overline{x}+Du)\right] = 0 \qquad (6c)$$

with boundary conditions (5b). The details of the solution of equations (6) are shown in appendix B.

Equations (6) are first solved for the control (u) as a function of $\overline{x}$ and $\overline{\lambda}$ from equation (6c) and then for the states and adjoint variables as a function of time from (6a) and (6b). The results of the solution to the control equation are given by (eq. (B4))

$$u = \frac{K}{2I}\lambda_2 - K_{\dot{\alpha}}x_2 - K_{\alpha}x_1 + K_m\omega \qquad (7)$$

The form of the control as shown in equation (7) has been modified from the direct solution of equation (6c) where the state x_3 has been rewritten in terms of the reaction wheel speed ω and the body angular rate x_2. The control in this form is more convenient for a practical problem because ω is directly measurable.

With the substitution of the control (u) into equations (6a) and (6b), $\overline{x}$ and $\overline{\lambda}$ can be found as functions of time and the initial and final boundary conditions on $\overline{x}$. The solution to this portion of the problem is given by equations (B7), (B9), and (B10). The parts of these equations required to form the control are

$$\left.\begin{array}{l} x_1(t) = \dfrac{K^2}{2RI^2}\left[\lambda_1(0)\dfrac{t^3}{6} - \lambda_2(0)\dfrac{t^2}{2}\right] + x_2(0)t + x_1(0) \\[4mm] x_2(t) = \dfrac{K^2}{2RI^2}\left[\lambda_1(0)\dfrac{t^2}{2} - \lambda_2(0)\,t\right] + x_2(0) \\[4mm] \lambda_2(t) = -\lambda_1(0)t + \lambda_2(0) \end{array}\right\} \qquad (8)$$

The adjoint variable boundary conditions are given by

$$\lambda_1(0) = \frac{12RI^2}{K^2T^2} \left[2x_2(0) + \frac{2}{T}(x_1(0)-x_1(f)) - (x_2(0)-x_2(f)) \right]$$

$$\lambda_2(0) = -\frac{12RI^2}{K^2T^2} \left[\frac{T}{3}(x_2(0)-x_2(f)) - (x_2(0)T+x_1(0)-x_1(f)) \right]$$

$$\left. \right\} \quad (9)$$

The trajectories given by equations (8) can now be substituted into equation (7) to generate the desired command signal for the transfer of the states.

Trajectory Control

Many ways could be developed for driving the vehicle along the prescribed path during the trajectory control portion of a scan. But for the scanning problem it seemed best to have the controller operate essentially the same during the trajectory control as during the end-point control in order to provide a smooth transition between the various segments of the scan. Therefore, the control form is taken to be the same as given by equation (7), which is repeated below.

$$u = \frac{K}{2I}\lambda_2 - K_\alpha x_1 - K_{\dot{\alpha}} x_2 + K_m \omega \qquad (7)$$

Since there is a prescribed trajectory, x_1 and x_2 are specified as a function of time, but λ_2 is unknown and must be specified as a function of the trajectory. The relationship between λ_2 and the state variables can be seen from equation (B6a)

$$\dot{x}_2 = -\frac{K^2}{2RI^2}\lambda_2 \qquad (10)$$

Equation (10) yields λ_2 as a function of $\dot{x}_2$, the acceleration of the vehicle along the prescribed path. The control equation for the path constraint problem then becomes

$$u = -\frac{RI}{K}\dot{x}_2 - K_\alpha x_1 - K_{\dot{\alpha}} x_2 + K_m \omega \qquad (11)$$

Error Analysis

Equation (7) is a command signal (u) to drive a satellite with a prescribed maneuver. The maneuver specification requires that the error between the desired trajectories and the actual vehicle motion be small; consequently, it is desirable to investigate the error behavior of the system during a maneuver. Because the form of the command is the same for both the end-point and trajectory controlled motions, one error analysis will do for

both. When the end points of a given segment are specified, the vehicle must follow a fixed trajectory; hence both commands actually become the trajectory type.

The error between the commanded and the actual vehicle motion can be defined by the following:

$$\epsilon = x_1 - x_{1_c} \tag{12}$$

The command equation can be rewritten in the form

$$u = -\frac{RI}{K}\ddot{x}_{1_c} - K_{\dot{\alpha}}\dot{x}_{1_c} - K_\alpha x_{1_c} + K_m\omega \tag{13}$$

Using the error definition and the command equation as written above with the plant equation (1) gives the following error equation:

$$\ddot{\epsilon} + \left(\frac{KK_{\dot{\alpha}}}{RI}\right)\dot{\epsilon} + \left(\frac{KK_\alpha}{RI}\right)\epsilon = \frac{T_e}{I} \tag{14}$$

This equation can be written in a more convenient form by use of the Laplace transform.

$$E(S) = \frac{\left(S + \frac{KK_{\dot{\alpha}}}{RI}\right)\epsilon(0) + \dot{\epsilon}(0) + \frac{T_e(S)}{I}}{S^2 + \frac{KK_{\dot{\alpha}}}{RI}S + \frac{KK_\alpha}{RI}} \tag{15}$$

The error shown by equation (15) depends only on the initial conditions of the error and error rate and on the external torques on the vehicle and not on the trajectory to be followed. Consequently, the command system developed simplifies the mechanization of a system considerably because the system dynamic characteristics may be set (by adjusting the feedback gains) while the vehicle is holding some fixed attitude or null position to give the desired error performance. If for example $\epsilon(0)$, $\dot{\epsilon}(0)$, and $T_e(S)$ are zero, the maneuver will be carried out without any dynamic error. Further, if a transient disturbance during a maneuver drives the system away from the desired path, the control will act to return the vehicle to this path.

Control System Operation

Once the complete trajectory has been defined, the control operates to follow the trajectory calculated by the controller. At a given time, the signal from the controller consists of terms proportional to the desired accelerations, rate, and position plus a term proportional to the back emf of the reaction wheel motor. This signal is summed with position and rate feedbacks generated within the plant to form the signal to the reaction wheel motor. If conditions are ideal, that is, there are no external disturbances, no initial errors in position and rate, and all the components are exactly as mathematically described, the position and rate command signals will be

equal and opposite to the feedback signals. Therefore, the input to the reaction wheel will be only the acceleration and back-emf signals. If, however, a position or rate error does exist, the position and rate feedback will cause an error signal which will sum with the acceleration command and drive the vehicle toward the prescribed trajectory.

The presence of a continuous acceleration command in the control scheme developed in this report is one of the principal differences between it and the scheme developed in reference 1. The requirement that the control elements be of the proportional type in the present study can be considered merely a matter of choice. If nonlinear elements were present in the position and rate error signals, as they are in the control developed in reference 1, the structure of the command signal would remain the same; that is, the acceleration and back-emf terms would remain unchanged, and the rate and position terms would cancel those generated by the plant under ideal conditions. For an AOSO type scanning motion, the operation of both controllers during the scan line motion is basically the same because no acceleration is required. The command signals differ when an acceleration is required, such as during the line change maneuver. In the control developed in reference 1, the input signal is shaped to saturate the input to the torque motor to cause maximum torque. Various shaping schemes are used to minimize the error at the start of the subsequent scan. In the present development, the transition between scan lines is simply a trajectory which requires an acceleration command. This acceleration command plus the tachometer feedback signal provide exactly the same input to the reaction wheel as the saturation command required in reference 1. The difference is that the saturation command employed in reference 1 is essentially an open-loop operation for which the system characteristics must be known precisely, particularly the variation of torque with applied voltage; whereas the command developed in this report includes a position and rate error signal which will drive the vehicle toward the desired trajectory.

EXPERIMENT

The control scheme was tested for a scanning motion similar to that of an AOSO. An air bearing table with reaction wheel torquers was used to simulate the spacecraft, and a digital computer was used for the controller. The geometry of the scan motion performed by the air bearing table (fig. 2) was not precisely the same as an AOSO scan because of hardware limitations.

The basic requirement of the scan motion is that the vehicle rotate about the yaw axis at a constant rate of 3.6 arc minutes/second over a given scan width (18 arc min for the experiment) while the pitch position is held to a particular value. The roll axis is to be held to zero at all times during the scan. For the motion along a scan line, the path is given as a function of time. At the end of a scan line the vehicle must reverse yaw rate and step to a new pitch position (the pitch change is 2 arc min). Only the end points and the time of transfer are specified for this portion of the scan, and both the pitch and yaw motion occur simultaneously and must be

completed in 4 sec. It was also necessary to include start and stop commands (assumed to be of the end-point type) which would move the vehicle into the scan from a zero position and return it to this position at the completion of the scan.

Experimental Equipment

Spacecraft simulator.- The air bearing table (simulating the spacecraft) with the various drive components onboard is shown in figures 3(a) and 3(b). This table is supported on a spherical gas bearing. The position reference for the table was obtained from the two optical sensors shown. Both were solid state photocell type detectors rigidly attached to the table. Pitch and yaw signals were obtained from one sensor and roll from the other. The size of the scan and the accuracy with which the scan could be performed was limited by the characteristics of the optical sensors. The linearity of the sensor output was the limit on the accuracy of the scan and was about ±0.5 arc minute. After one stage of amplification on the table, the sensor output signals were sent, through the wires shown on figure 3, to the digital computer through analog to digital (A/D) converters.

The torquers were heavy metal flywheels driven by dc torque motor drivers. The drive signal for the torquers was generated in the digital computer, converted to an analog signal by a D/A converter, and sent to the torque motors through dc power amplifiers. No tachometer loop was used with the motors. The motor characteristics were such that the back-emf effects were small and the lack of a cancelling signal in the command had little effect.

System damping was provided by a rate gyro package which measured the angular rates of the table about the three control axis. (The use of gyros for damping was assumed in the theoretical development.) The position signals were sent to the computer for error summation but the rate signals were not. An initial attempt to sum the rate signals in the computer resulted in poor performance because noise on the gyro output caused difficulties with the A/D conversion of the signal. To avoid the problem, the rate signal was fed directly to the dc power amplifiers on the air bearing table and summed at that point with the computer output. Electrical signals between the air bearing table and the computer were transmitted through fine wires to the table from directly overhead. These wires also supplied electric power to the table. Since the angle motion of the scan was small and the wires were very fine, the torques were negligible and the wires provided a great convenience because they eliminated the need for batteries and telemetry equipment.

The following is a list of the various inertias of the table and reaction wheels and of the motor characteristics. For the air bearing table, the inertias about each of the control axes were different, but the reaction wheel characteristics were all the same.

Air bearing table.

I_{roll} = 39 kg-m^2

I_{pitch} = 26 kg-m^2

I_{yaw} = 71 kg-m^2

Reaction wheels.

J = 0.011 kg-m^2

Stall Torque = 0.28 N-m

Time Constant = 15 sec

Max. rpm = 400 rad/sec

Computer.- All of the controller logic for the command signal generation and the error summation (with the exception of the rate signal) was in the digital computer and the computer acted as an on-line real-time controller. The computer used for the experiment was an SDS 920. Figure 4 is a block diagram of the computer operations for the yaw axis (the pitch axis operations are similar).

The computer was programmed with a combination of Fortran and symbol language. The command signal generation and the error summation were in Fortran. The input-output routines used symbol language. Fortran was selected so that the program could be modified easily. It was frequently necessary during the tests to change the gain constant to match the command signal characteristics to those of the table to minimize performance error. For the operation of the scanning motion, the frequency with which the computer cycled through the program from input to output was about 12 Hz. To avoid a stability problem it was necessary to keep the natural frequency of motion about each of the table axes below 1 Hz. The effect on testing was to limit the feedback gains and hence the minimum error of the system.

Test Procedures and Results

One problem of the experiment was the matching of the command signal to the characteristics of the various components on board the air bearing table in order to optimize the table performance. Optimization for these particular test runs is taken to mean adjusting the input command shape so that the difference between the commanded trajectory and the actual trajectory was as small as possible. The adjustable quantities on the input command were the values of the feedback gains and the motor torque gains. Initial values of these gains were computed from calibration data on the various components and were then varied as necessary for the best performance.

12

Typical outputs for two runs after the gains had been set are shown in figures 5 and 6. Figures 5(a) and 6(a) present time histories for a scan motion that went once around the scan loop and then returned to zero, and figures 5(b) and 6(b) present similar data for a scan that went twice around the loop. In figure 5 plots of yaw and pitch motion show the basic geometry of the scan. Figure 6 is a strip chart recording of the motion of the air bearing table as a function of time. The traces in figures 5 and 6 are outputs from the computer D/A converters. In addition to signal noise, the traces show the quantization of the digital computer.

At the start of a scan it was necessary to have the reaction wheels actively controlling the table on all three axes and holding a zero position. The zero in this case was not a specified geometric position, but was wherever the table was oriented. While this zero position was held the computer function was to close the position loops for the three axes and to generate the necessary drive signals for the reaction wheel motors. The command input (u) was zero at this time. The position feedback gains and the system damping were then set to hold zero as closely as possible. The cycle time of the digital computer imposed a limit on the position feedback gains and therefore limited the ability of the system to keep the attitude within a given error. With the gains at the maximum practical value, the system natural frequency, from observation of the position error signal, was about 1 Hz. This performance of the air bearing table can be seen by the traces on the left end of figure 6 where time was taken to be zero at the beginning of the scan motion. During the zero hold phase the control system held the table within about ±10 arc sec. This figure is only a qualitative measure of the operation of the control system. The sensor output is only linear to ±0.5 arc min, and hence measurements below this figure are only approximate.

A scan motion was started by having the computer generate the required command signal to sum into the feedback loop. During the scan all the gains and control loop characteristics remained as they had been for the zero hold operation. The response of the air bearing table to the commanded trajectory is shown by the traces of actual pitch and yaw position. As can be seen from figure 6, the motion of the table duplicated the required motion very well. The yaw and pitch error traces show the difference between the commanded position and the actual position. Ideally, the error curve should be the same whether or not the table is scanning or holding zero. The error traces show that the control scheme performed quite well in this respect. The pitch axis in particular held very nearly the same error level during the scan as during the zero hold operation. The yaw axis performance was also good but some variation of the error was caused by the scan motion. This error variation is directly related to the yaw position and repeats throughout the scan, and is due to a nonlinearity in the yaw sensor output signal. The table control system was unable to track the resulting signal without an error. The obvious solution to the problem is a sensor with better output characteristics, such as a gimbaled device with a high precision pick off. Despite this sensor problem, the control system was able to hold the yaw error to less than ±30 arc sec.

CONCLUDING REMARKS

In the preceding analysis, a command scheme has been developed which will permit the precise control of a satellite performing a predetermined maneuver. The results of the analysis showed that a satisfactory control scheme would be to use a proportional type controller to drive the vehicle along the desired trajectory with zero error. The solution of the problem used the variational form of optimal control to provide the control input for the maneuver segment where only the end points of the maneuver and time of transfer are specified. To provide a smooth transition between segments of a maneuver, this same form of control was used for the maneuver segment where the trajectory to be followed was given. The control included feedback loops to drive the vehicle back onto the desired trajectory in the case of external disturbance or initial errors.

The theoretical control scheme was tested for a three axis control system with an air bearing table to simulate the spacecraft. The table was driven by reaction wheel torquers and had optical sensors and rate gyros for the position and rate measurements, respectively. The theoretical command input to the air bearing table was formulated from the analysis programmed on a digital computer. The digital computer acted as a signal generator for the command and also closed the control loops for the vehicle control system, and was therefore an on-line, real-time controller for the air bearing table. The maneuver prescribed for the table to perform was a two line raster scan which was a simplification of that for an AOSO. The test results showed that the control scheme did force the table to perform the required maneuver and that the error in tracking the desired trajectory was small.

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PLANT EQUATIONS

The plant to be controlled is composed of the vehicle inertia, the reaction wheel torquer, and the position and rate feedback. For one axis of the vehicle plus the reaction wheel, the total angular momentum with respect to inertial space, expressed in the vehicle coordinate system, is

$$H = I\Omega + J(\Omega+\omega) \tag{A1}$$

where I is the total moment of inertia of the vehicle without the reaction wheel, J is the reaction wheel moment of inertia, and ω is the wheel speed measured relative to the vehicle. Differentiating equation (A1) with respect to time and equating the momentum change to the external torques, with the assumption of small angular motion ($\Omega \approx \dot{\alpha}$), we obtain the equation of motion

$$T_e = I\ddot{\alpha} + J(\ddot{\alpha}+\dot{\omega}) \tag{A2}$$

The reaction wheel is driven by a dc motor and its acceleration when current is applied is given by

$$Ki = J(\ddot{\alpha}+\dot{\omega}) \tag{A3}$$

The current to the motor is related to the applied voltage by equation (A4). The applied voltage is taken to be the sum of the position and rate feedback, and an input command (u). The term $K_m\omega$ is the back emf of the motor.

$$e = iR + K_m\omega = K_\alpha\alpha + K_{\dot{\alpha}}\dot{\alpha} + u \tag{A4}$$

It has been assumed that the feedback gains in equation (A4) are linear. This assumption is, in general, not required but makes the analysis simpler. The only requirement for the solution is that the feedback characteristics be known.

The input current to the reaction wheel motors may be expressed as

$$i = \frac{1}{R} (K_\alpha\alpha+K_{\dot{\alpha}}\dot{\alpha}+u-K_m\omega) \tag{A5}$$

Equation (A2) combined with equations (A3) and (A5) gives the following set of equations for describing the rotational motion of one axis of the plant.

$$T_e = I\ddot{\alpha} + J(\ddot{\alpha}+\dot{\omega}) \tag{A6a}$$

$$\frac{K}{R} (K_\alpha\alpha+K_{\dot{\alpha}}\dot{\alpha}+u-K_m\omega) = J(\ddot{\alpha}+\dot{\omega}) \tag{A6b}$$

APPENDIX B

END-POINT CONTROL SOLUTION

The required steps for solving the end-point control problem are to solve equation (6c) for (u) as a function of the state variables $(\bar{x})$ and the adjoint variables $(\bar{\lambda})$. The control (u) is then substituted into equations (6a) and (6b) and the resulting set of differential equations solved for $(\bar{x})$ and $(\bar{\lambda})$ as a function of time and the boundary conditions on $(\bar{x})$.

To solve the control equation it is necessary to know the cost-function integrand in terms of the problem variables. Using equations (5c) and (A5)

$$L = \int_0^T \frac{1}{R} \left[K_\alpha x_1 + K_{\dot{\alpha}} x_2 + u - \frac{K_m}{J} x_3 + \frac{K_m}{J}(I+J)x_2 \right]^2 dt \qquad (B1)$$

Equation (4) and the integrand of equation (B1) may now be substituted into equation (6c) to solve for (u).

$$u = \frac{K}{2I} \lambda_2 - K_\alpha x_1 - \left(K_{\dot{\alpha}} + K_m \frac{I+J}{J} \right) x_2 + \frac{K_m}{J} x_3 \qquad (B2)$$

The control (u) is thus a function of one of the adjoint variables λ_2 and all three of the state variables. Of these four variables, only (x_3), the constant angular momentum, is known. In order to use the control it will be necessary to solve for the adjoint variable, λ_2, and the trajectory to be followed as a function of the end points and the time of control.

Equation (B2) can be simplified if it is noted from the angular momentum equation that

$$K_m \omega = \frac{K_m}{J} x_3 - K_m \frac{I+J}{J} x_2 \qquad (B3)$$

Substituting equation (B3) into equation (B2) gives

$$u = \frac{K}{2I} \lambda_2 - K_\alpha x_1 - K_{\dot{\alpha}} x_2 + K_m \omega \qquad (B4)$$

This equation shows that the desired control is composed of four terms, two of which effectively cancel the position and rate feedback as far as the reaction wheel input is concerned. The remaining terms include one which can be considered a tachometer feedback to match the voltage of the back emf and one which is due to an unknown term λ_2.

The next step is to solve equations (6a) and (6b) for $\bar{x}$ and $\bar{\lambda}$ as functions of time. They will be given in terms of the initial and final

boundary conditions on $(\bar{x})$. If the indicated operations are performed on equation (6b), the following equations result:

$$\dot{\lambda}_1 = \frac{KK_\alpha}{RI}\lambda_2 - 2K_\alpha\left[\frac{K_\alpha}{R}x_1 + \left(\frac{K_{\dot{\alpha}}}{R} + \frac{K_m}{R}\frac{I+J}{J}\right)x_2 - \frac{K_m}{JR}x_3 + \frac{u}{R}\right]$$

$$\dot{\lambda}_2 = -\lambda_1 + \left(\frac{KK_{\dot{\alpha}}}{RI} + \frac{KK_m}{RI}\frac{I+J}{J}\right)\lambda_2 - 2\left(K_{\dot{\alpha}} + K_m\frac{I+J}{J}\right)\left[\frac{K_\alpha}{R}x_1\right.$$

$$\left. + \left(\frac{K_{\dot{\alpha}}}{R} + \frac{K_m}{R}\frac{I+J}{J}\right)x_2 - \frac{K_m}{JR}x_3 + \frac{u}{R}\right]$$

$$\dot{\lambda}_3 = -\frac{KK_m}{RIJ}\lambda_2 + \frac{2K_m}{J}\left[\frac{K_\alpha}{R}x_1 + \left(\frac{K_{\dot{\alpha}}}{R} + \frac{K_m}{R}\frac{I+J}{J}\right)x_2 - \frac{K_m}{JR}x_3 + \frac{u}{R}\right]$$

$$\left.\right\}\quad\text{(B5)}$$

If the control (u) given by equation (B2) is now substituted into equations (B5) and (4), the plant and adjoint equations reduce to the following simple form:

Plant equations
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\left(\frac{K^2}{2RI^2}\right)\lambda_2 \qquad\qquad \left.\right\}\quad\text{(B6a)}$$
$$\dot{x}_3 = 0$$

Adjoint equations
$$\dot{\lambda}_1 = 0$$
$$\dot{\lambda}_2 = -\lambda_1 \qquad\qquad \left.\right\}\quad\text{(B6b)}$$
$$\dot{\lambda}_3 = 0$$

The effect of the control has been to reduce the plant equations to those of a simple rigid body with a torque proportional to λ_2. The adjoint equations are also simple and can be readily integrated and substituted into the plant equations to give the desired trajectories. Solving the adjoint equation gives

$$\lambda_1 = \lambda_1(0) \qquad\qquad\qquad\qquad\qquad\text{(B7a)}$$

$$\lambda_2 = -\lambda_1(0)t + \lambda_2(0) \qquad\qquad\qquad\text{(B7b)}$$

$$\lambda_3 = \lambda_3(0) \qquad\qquad\qquad\qquad\qquad\text{(B7c)}$$

with the unknown boundary conditions $\lambda_i(0)$. Substituting equations (B7) into (B6a) gives

$$\dot{x}_1 = x_2 \tag{B8a}$$

$$\dot{x}_2 = -\left(\frac{K^2}{2RI^2}\right)\left[-\lambda_1(0)t + \lambda_2(0)\right] \tag{B8b}$$

$$\dot{x}_3 = 0 \tag{B8c}$$

Integrating these equations gives

$$x_1 = \frac{K^2}{2RI^2}\left[\lambda_1(0)\,\frac{t^3}{6} - \lambda_2(0)\,\frac{t^2}{2}\right] + x_2(0)t + x_1(0) \tag{B9a}$$

$$x_2 = \frac{K^2}{2RI^2}\left[\lambda_1(0)\,\frac{t^2}{2} - \lambda_2(0)t\right] + x_2(0) \tag{B9b}$$

$$x_3 = x_3(0) = H \tag{B9c}$$

Both the initial and the final boundary conditions for the state variables and the control time T to the end point $\bar{x}(T)$ are known for equations (B9). Hence, the initial conditions on the adjoint variables can be found.

$$\left.\begin{aligned}
\lambda_1(0) &= \frac{12RI^2}{K^2T^2}\left[2\,x_2(0) + \frac{2}{T}\,(x_1(0)\text{-}x_1(f)) - (x_2(0)\text{-}x_2(f))\right] \\[2ex]
\lambda_2(0) &= -\,\frac{12RI^2}{K^2T^2}\left[\frac{T}{3}\,(x_2(0)\text{-}x_2(f)) - (x_2(0)T + x_1(0)\text{-}x_1(f))\right]
\end{aligned}\right\} \tag{B10}$$

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1. Edwards, Fredrick G.: Use of Reaction Wheels on Non-Linear Control for a Satellite Scanning a Small Celestial Area. NASA TN D-3592, 1966.

2. Athans, M; and Falb, P. L.: Optimal Control; An Introduction to the Theory and Its Application. McGraw-Hill, 1966.
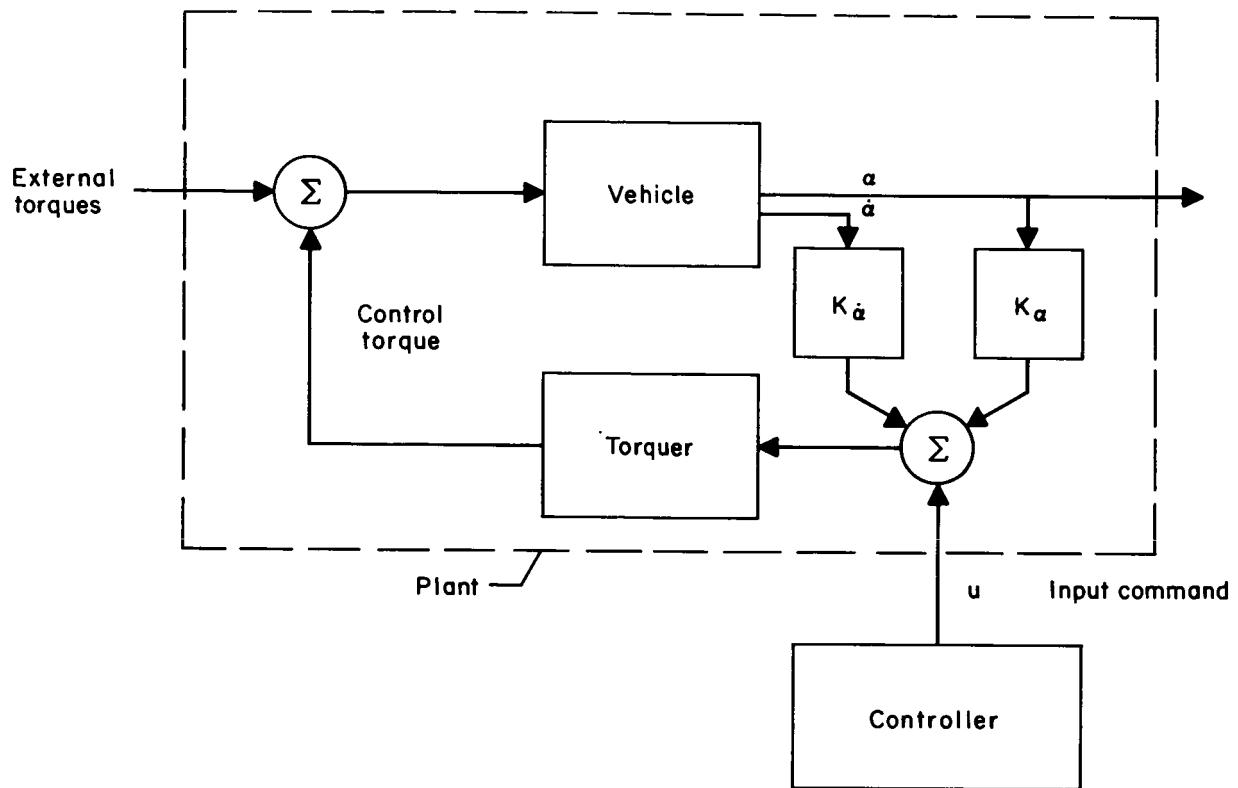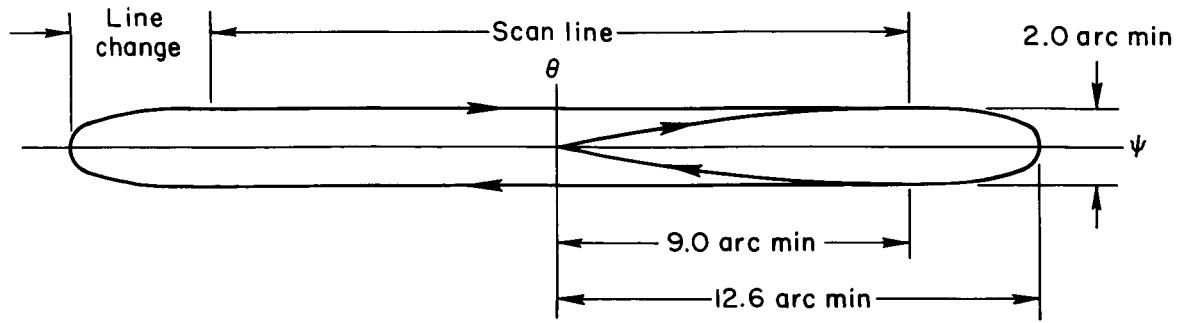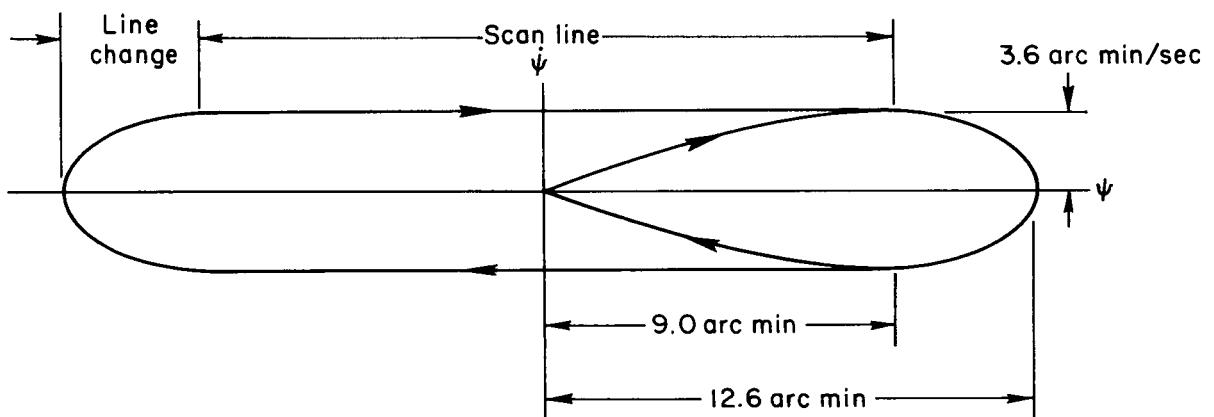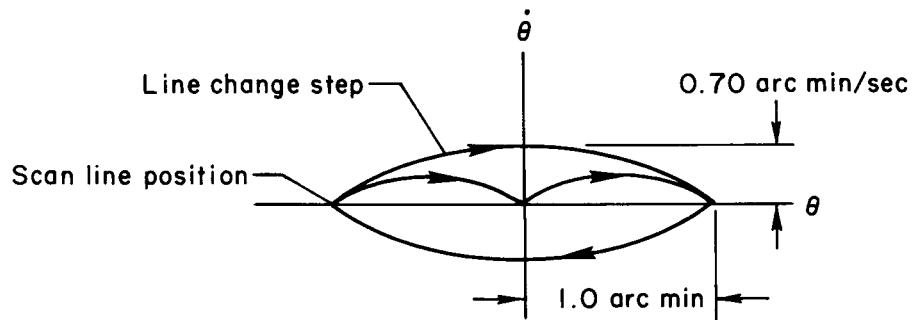
Figure 1.- Block diagram of plant and controller system.

(a) Plot of yaw and pitch motion.
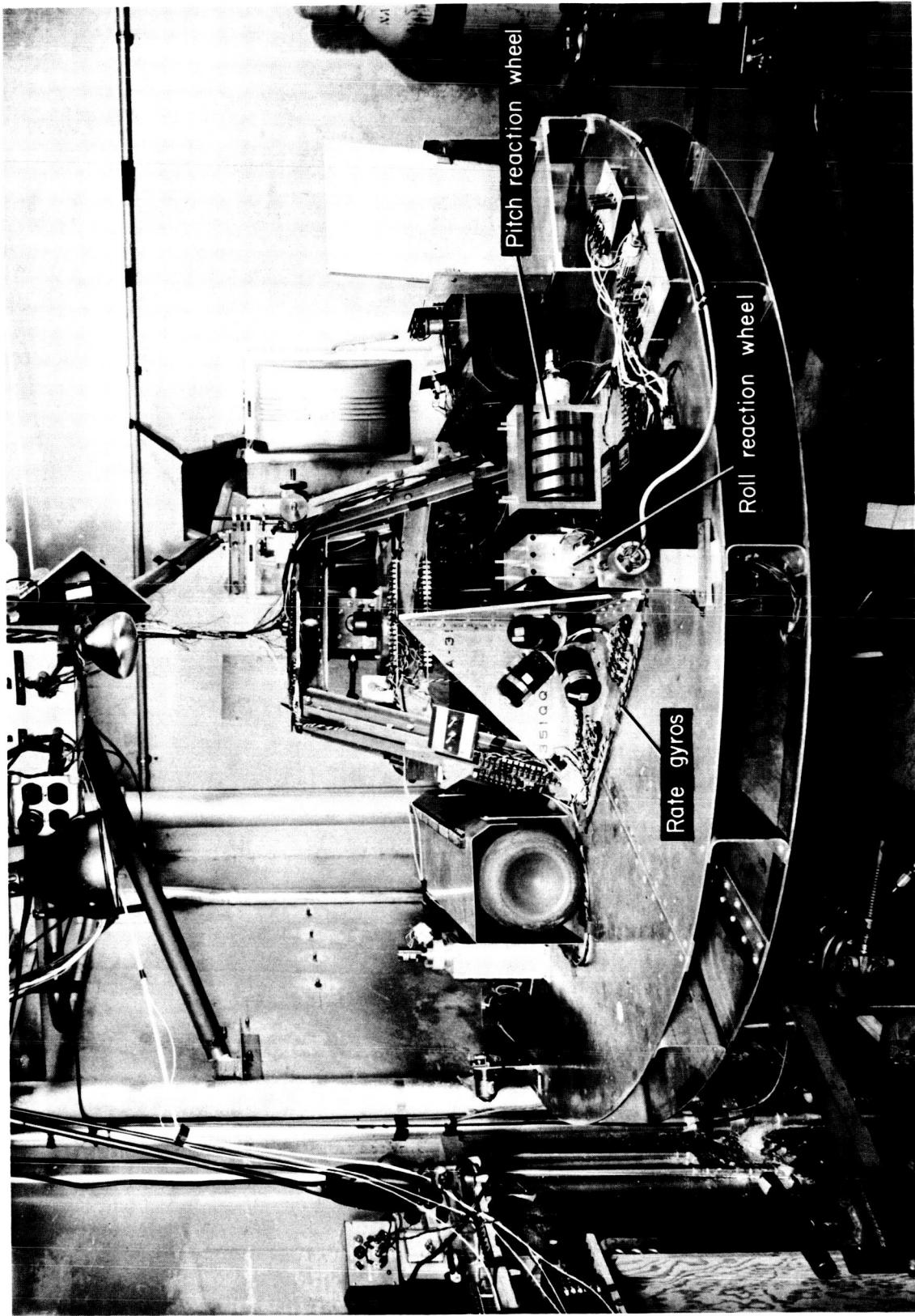
(b) Phase plane plot of yaw motion.


(c) Phase plane plot of pitch motion.

Figure 2.- Plots of scan motion.
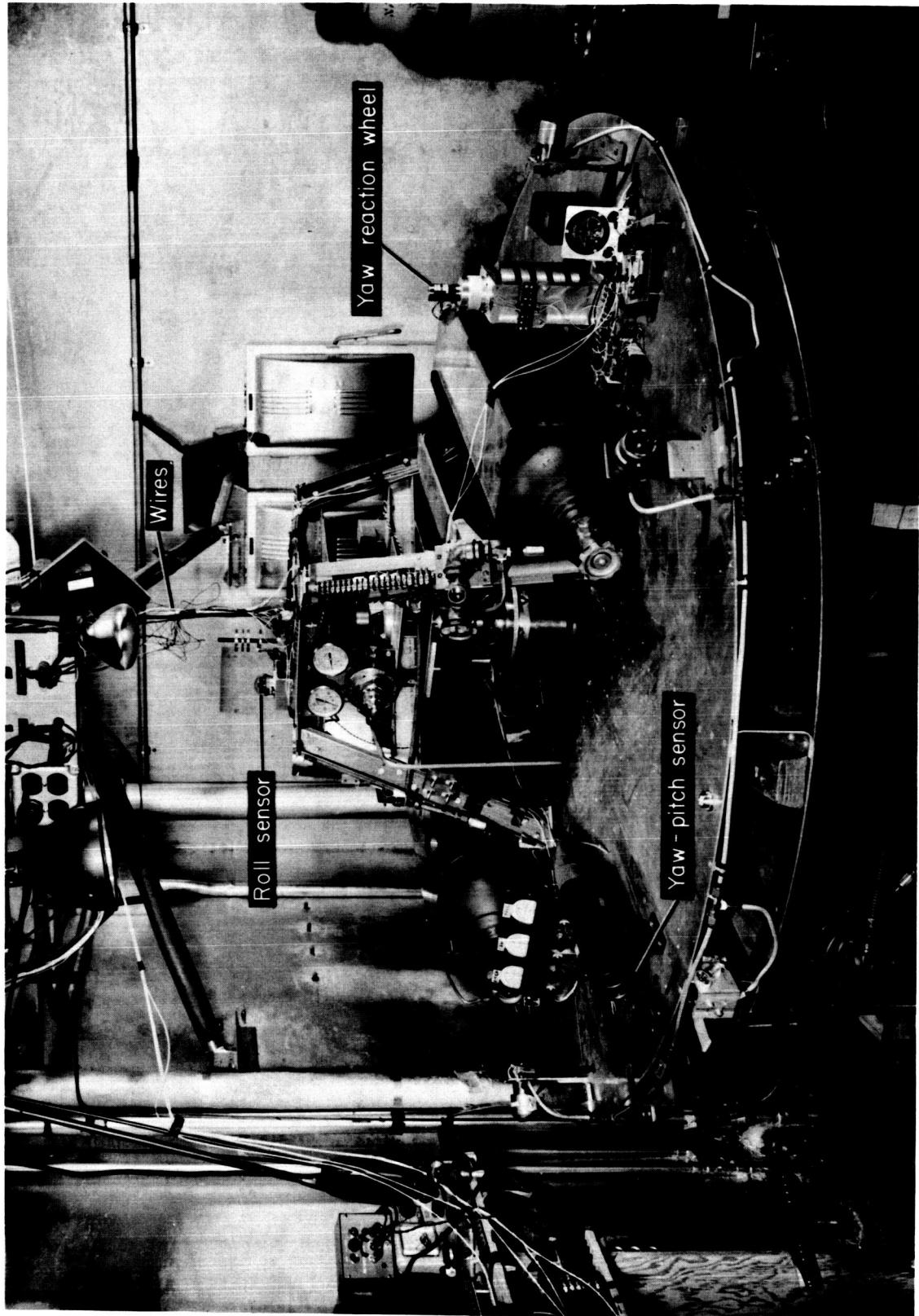
(a) Front view of air bearing table.

Figure 3.- Photograph of air bearing table installation.

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A-36821.1

(b) Rear view of air bearing table.

Figure 3.- Concluded.
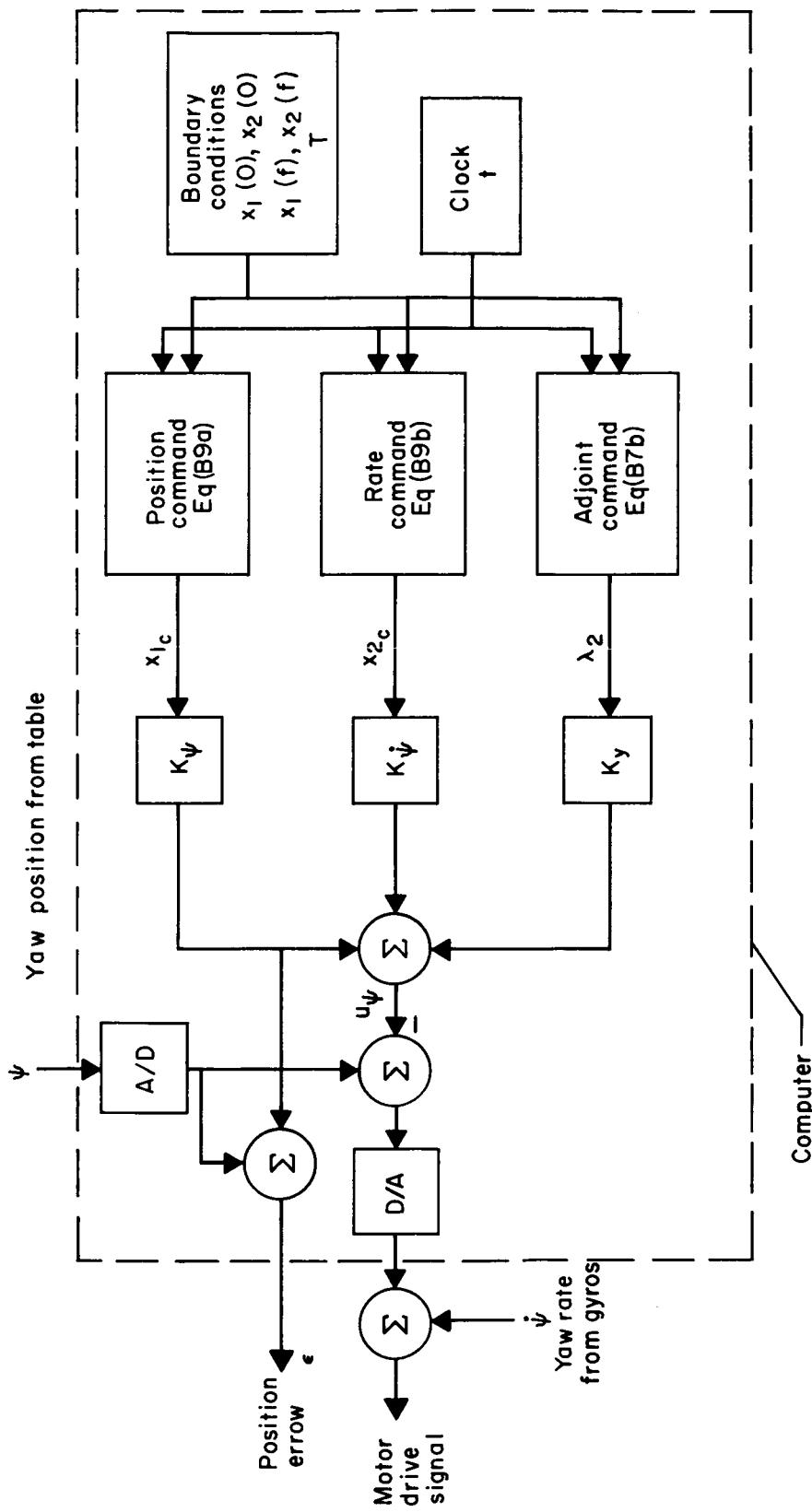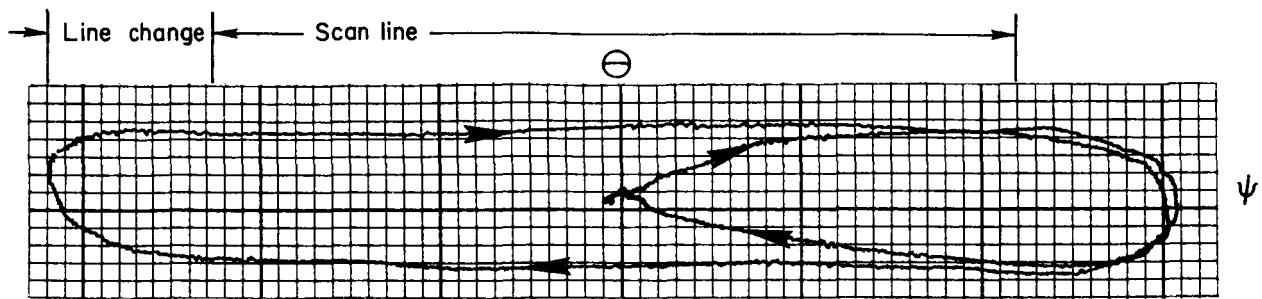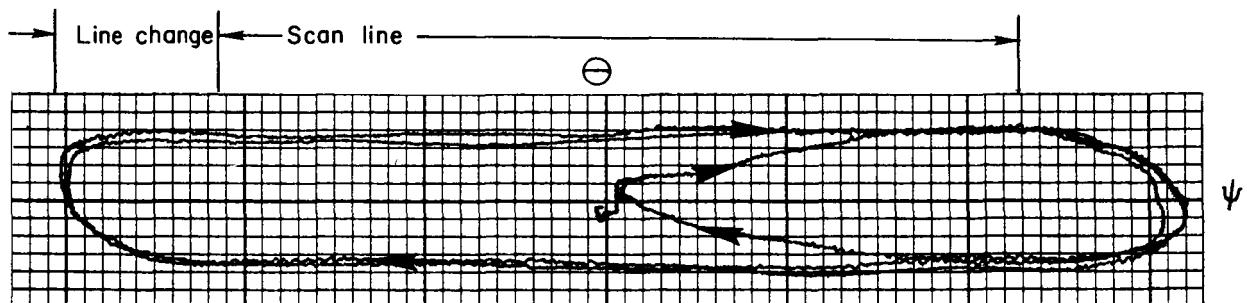
Figure 4.- Block diagram of computer operations, yaw axis.
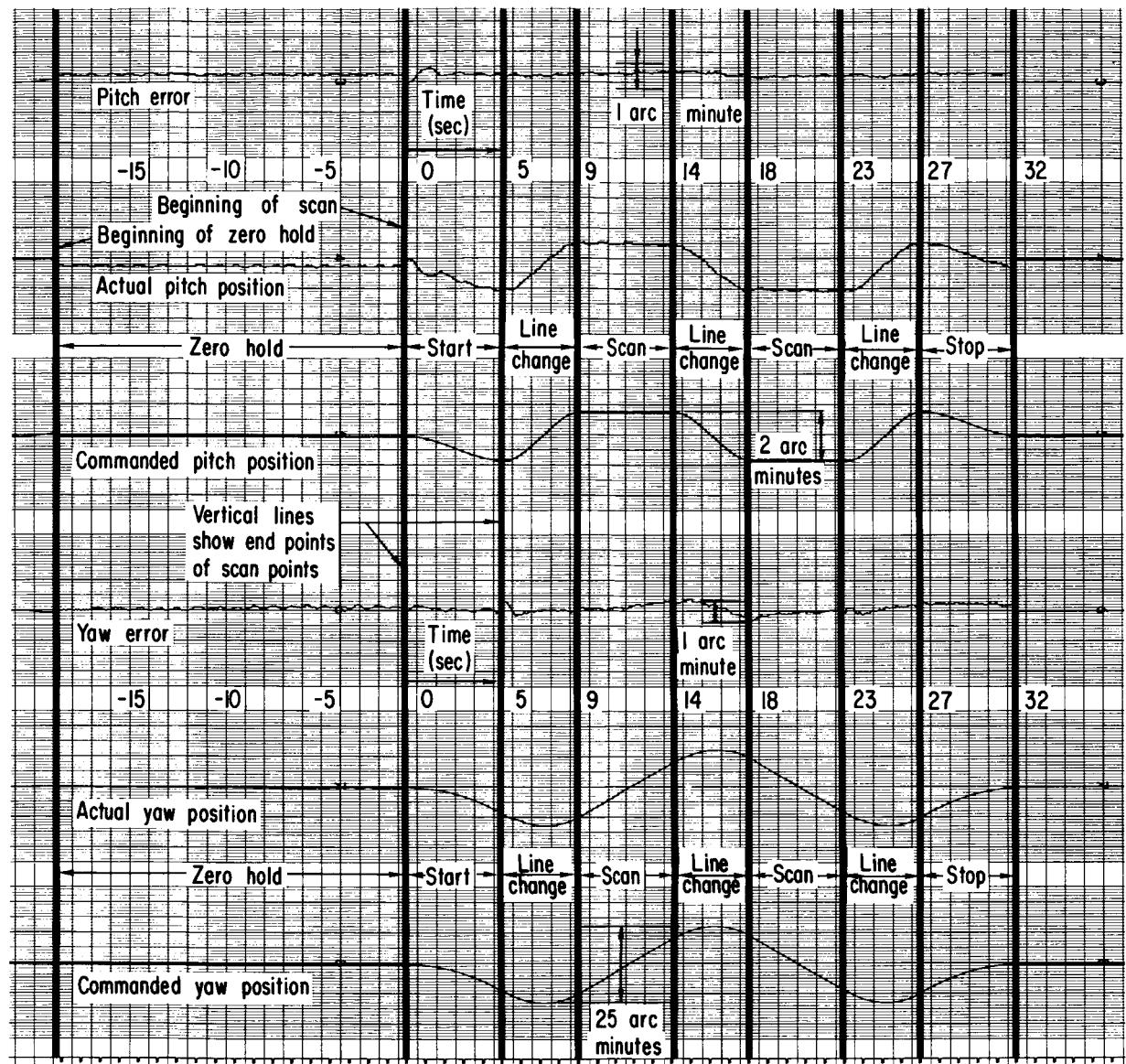
(a) Scan motion; one loop.
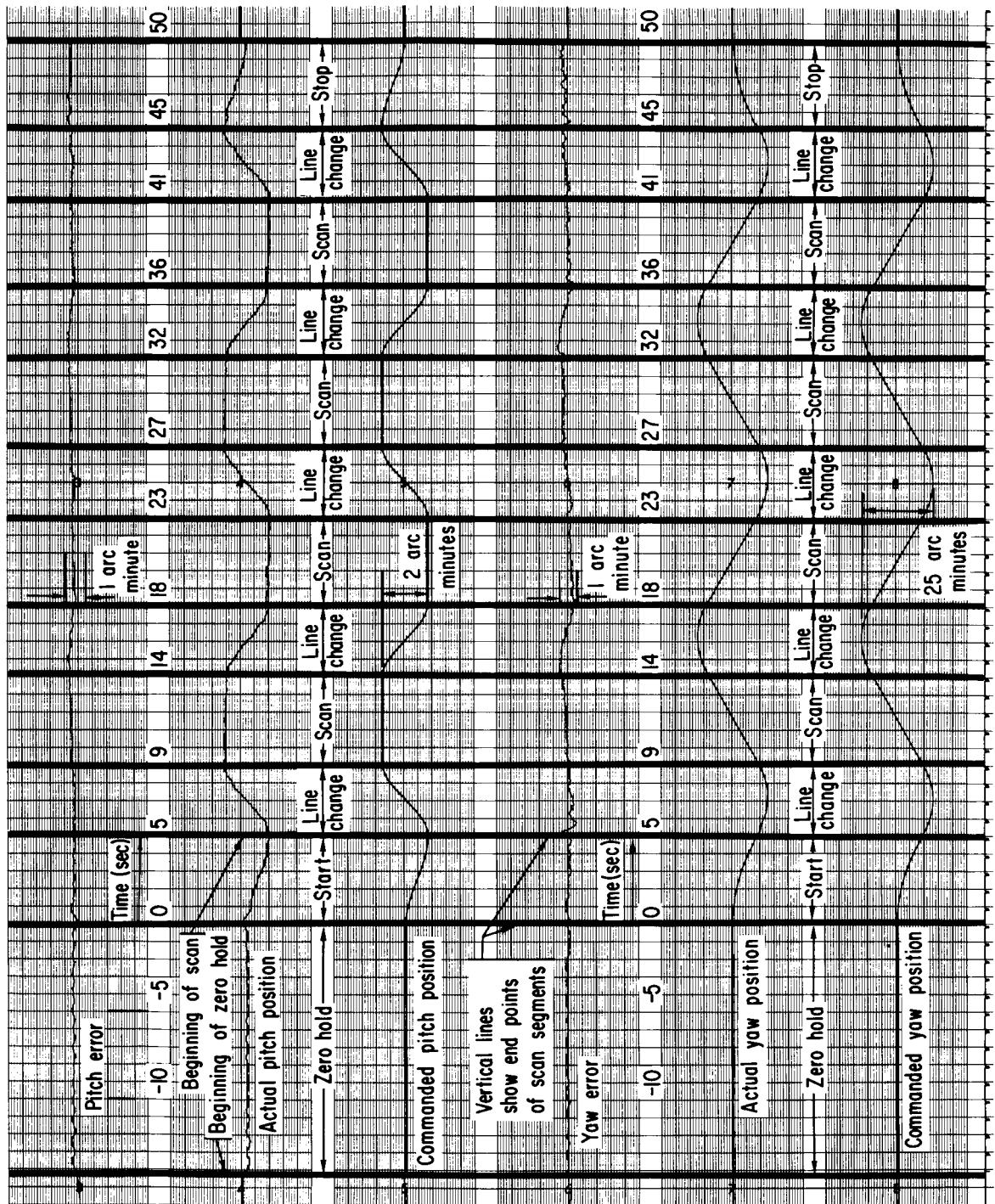
(b) Scan motion; two loops.

Figure 5.- Real-time record of yaw and pitch position during a scan run.

(a) Scan motion, one loop.

Figure 6.- Strip chart recording of scan motion.

(b) Scan motion, two loops.

Figure 6. - Concluded.